

Mathematics of Fluids and Plasmas

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TRUST

Is it a solid or a fluid?

- Solid $\rightarrow$ liquid $\rightarrow$ gas $\rightarrow$?
- Heat a gas (thousands or millions $^{\circ}\text{C}$) $\Rightarrow$ plasma
- Air flows around a room
- Water flows in a river
- Metal objects don't flow.
- Fluids: gases, liquids, plasma.
- Is honey a fluid? ✓ You can pour it, it's just very slow.
- Is toothpaste a fluid? ✓ a special kind

Fluids on Earth

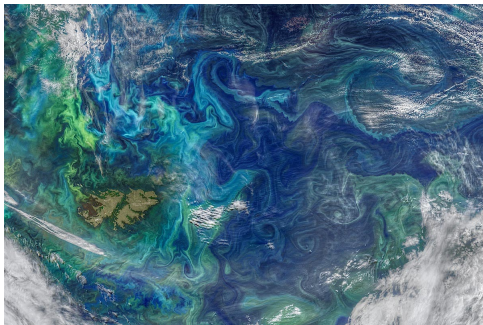


Figure: Ocean eddies. Credit: NASA/Goddard Space Flight Center Ocean Color/NOAA-20/NASA-NOAA Suomi NPP



Figure: Niyiragongo Volcano, Democratic Republic of the Congo. Credit: Victoria Milanez Fernandes

Plasma and instabilities in nature



Figure: Clouds. Credit: UCAR



Figure: Lightning in Tomsk.
Credit: Mikhail Arshinov

The Sun: ball of plasma

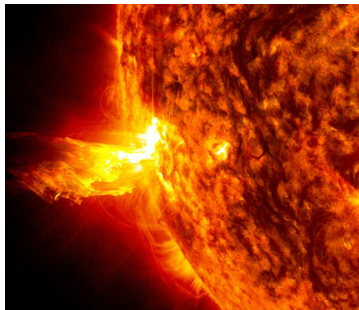


Figure: Solar flare. Credit: NASA Goddard

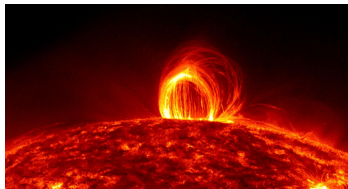


Figure: Plasma rain on the Sun.
Credit: GODDARD SCIENTIFIC
VISUALIZATION
STUDIO/SDO/NASA

Aurora: Plasma in the Atmosphere



Figure: Aurora in Sweeden.
Credit: Henrik Drake
(distributed via imagegeo.egu.eu)

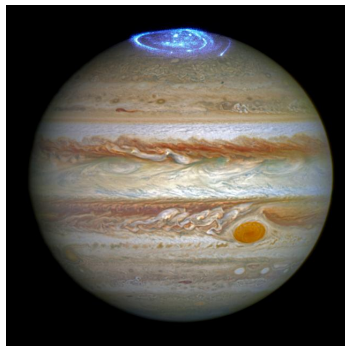


Figure: Aurora on Jupiter.
Credit: Credits: NASA, ESA,
and J. Nichols (University of
Leicester)

Creating Plasma on Earth

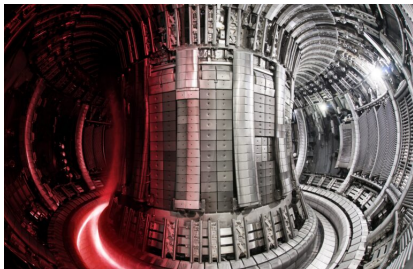


Figure: Inside Joint European Torus (JET) with superimposed plasma Credit: UKAEA, curtesy of EUROfusion

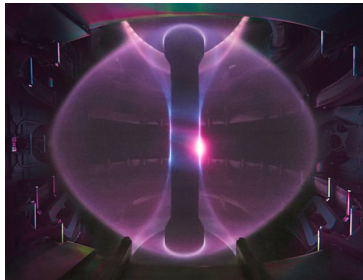


Figure: Plasma inside Mega Ampere Spherical Tokamak (MAST) Credit: UKAEA

What's this got to do with maths?

- Atmospheres, clouds, volcanoes, oceans, stars, fusion . . .
- Sounds like a lot of physics. . .
- But equations describe everything in the Universe!
- Physics just informs the maths.
- We will learn the main equations in this course.

What is this course about?

- Some of these fluids are very interesting but complex.
- Let's start with an introductory course.
- Relevant if you're project is in / or you are simply interested in these fields:

Hydrodynamics

hydro - water-like,

dynamics - how and why it moves around and changes.

Magnetohydrodynamics

- Plasmas are electrically charged,
- They are affected by magnetic fields,
- + magneto - magnetic

What you need:

- There are lots of vectors $\Rightarrow$ several variable calculus, linear algebra.
- It is not necessary to have studied physics, just bring some common sense and intuition!
- There are recommended textbooks for additional reading if you are interested.

What we will learn:

2 parts - hydrodynamics and magnetohydrodynamics.

For each part we use maths to describe fluid:

- motion,
- waves (ocean, sound, plasma waves),
- phenomena like the Kelvin Helmholtz Instability $\Rightarrow$



Figure: Clouds. Credit: UCAR